Improvement of Photon Counting by Means of a Pulse Height Analyzer*

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By taking into account the pulse height distribution, the maximum available information from a photomultiplier is examined in the case of very low light intensities and is compared with that information which is obtained in the conventional photon counting method using an optimized triggering level. Using the proposed method, the relative error is diminished by about a factor 1.4, i.e., nearly a factor of 2 is gained in the measuring time in the case of an EMI photomultiplier 6256.

THE detectivity of a photomultiplier, used to count single photons, is given by the ratio of its photosensitivity to dark noise. In the conventional photon counting technique, this ratio can be influenced only by changing the integral triggering level. It is well known[1] that the pulse height distribution of the dark noise $n_0(U)$ (U=pulse height) differs from that of the photoeffect-sensitivity $\epsilon(U)$. These two distributions differ considerably, the more so as thermionic emission from the cathode is suppressed by cooling. In the conventional counting technique an optimum lower triggering level U_0 is determined from these pulse height distributions $\epsilon(U)$ and $n_0(U)$ in the case of very small signals by the equation

$$(\partial/\partial U_0)\left[\int_{U_0}^{\infty}\epsilon(U)dU\right]\Big/\left\{\left[\int_{U_0}^{\infty}n_0(U)dU\right]^{\frac{1}{2}}\right\}=0. \quad (1)$$

Since only little improvement results in using an upper triggering level, it is usually omitted. Hence, this way of counting leads to a loss of information available from the multiplier. On the one hand, the sensitivity below U_0, which is omitted in the usual counting technique, is not zero; on the other hand, the dark pulse counting rate above U_0 is generally not proportional to the counting rate from photoelectrons. Even if a better signal-to-noise (S/N) ratio exists in a certain range of pulse height, it cannot be taken advantage of using the conventional integrating method. In the following we describe an optimization procedure for the case of very low signal intensities. By taking into account the true photoelectron counting rate and the dark pulse counting rate as functions of the pulse height, we make optimum use of the information given by the multiplier. The results will be verified empirically.

Dividing the range of the pulse heights into N channels i—for example, by use of a pulse height analyzer—we may get from each channel separately a value M_i for the incident photon rate I. This value M_i is determined by the total count N_i in the channel i and by the dark pulse count N_{0i} of the same channel,

$$M_i=(N_i-N_{0i})/\epsilon_i, \quad (2)$$

where ϵ_i, the photosensitivity, is the number of pulses in channel i created per incident photon.

N_{0i} is given by the equation

$$N_{0i}=n_0(U_i)\cdot\Delta U\cdot t, \quad (3)$$

where t is the measuring time, ΔU is the channel width, and $n_0(U_i)$ is the dark count rate at U_i per unit height. The ideal value M_i should be equal to $I\cdot t$.

The relative error of the measured value M_i is

$$\Delta M_i/M_i=(N_i+N_{0i})^{\frac{1}{2}}/(N_i-N_{0i}),$$

or approximately

$$\Delta M_i/M_i=(2N_{0i})^{\frac{1}{2}}/(N_i-N_{0i})$$

if $N_i-N_{0i}\ll N_{0i}$.

A linear combination (weighted average) of the values M_i in the single channels,

$$M=\sum_{i=1}^{N}a_iM_i \quad \text{with} \quad \sum_{i=1}^{N}a_i=1,$$

actually represents the measured quantity M. Here, M does not depend on a_i, and the optimization is reduced to the problem of minimizing the error by a suitable choice of a_i. The mean error ΔM of the measured quantity M is given by the quadratic sum of the individual errors, because these are independent statistical errors,

$$(\Delta M)^2=\sum_{i=1}^{N}(a_i\cdot\Delta M_i)^2=2\cdot t\cdot\sum_{i=1}^{N}\frac{a_i^2\cdot n_{0i}}{\epsilon_i^2}, \quad (4)$$

where n_{0i} is written for $n_0(U_i)\cdot\Delta U$.

Since the minimum of the error is also the minimum of its square, the following set of equations has to be solved:

$$\frac{\partial}{\partial a_i}\sum_{i=1}^{N}\frac{a_i^2n_{0i}}{\epsilon_i^2}=0, \quad \sum_{i=1}^{N}a_i=1.$$

We arrive at a solution with the aid of a Lagrange multiplier λ

$$\frac{\partial}{\partial a_i}\left[\sum_{i=1}^{N}\frac{a_i^2n_{0i}}{\epsilon_i^2}+\lambda\left(\sum_{i=1}^{N}a_i-1\right)\right]=0. \quad (5)$$

From Eq. (5) there results

$$2(a_i n_{0i}/\epsilon_i^2)+\lambda=0;\quad a_i=-(\lambda\epsilon_i^2/2n_{0i}). \tag{6}$$

With the boundary condition that

$$\sum_{i=1}^{N} a_i=1,$$

there follows

$$\sum_{i=1}^{N}\frac{\lambda\epsilon_i^2}{2n_{0i}}=-1;\quad \lambda=-2\bigg/\sum_{i=1}^{N}\frac{\epsilon_i^2}{n_{0i}}, \tag{7}$$

and therefore

$$a_i=\epsilon_i^2\bigg/n_{0i}\cdot\sum_{i=1}^{N}\frac{\epsilon_i^2}{n_{0i}}. \tag{8}$$

For the optimum value M one obtains

$$M=\sum_{i=1}^{N}(N_i-N_{0i})\epsilon_i\bigg/n_{0i}\cdot\sum_{i=1}^{N}\frac{\epsilon_i^2}{n_{oi}}$$

or

$$M=\sum_{i=1}^{N}(N_i-N_{0i})\frac{\epsilon_i}{n_{oi}} \tag{9}$$

because the sum $\sum^{N}{}_{i=1}(\epsilon_i^2/n_{0i})$ may be included in the calibration for the measurement. The relative error is determined as

$$\frac{\Delta M}{M}=(2)^{\frac{1}{2}}\bigg/I\left[t\sum_{i=1}^{N}\frac{\epsilon_i^2}{n_{0i}}\right]^{\frac{1}{2}}. \tag{10}$$

By use of the conventional counting technique the relative error would be

$$\Delta M'/M'=(2\sum_{i=z}^{N}n_{0i})^{\frac{1}{2}}\bigg/\sum_{i=z}^{N}\epsilon_i\cdot I\cdot t^{\frac{1}{2}}, \tag{11}$$

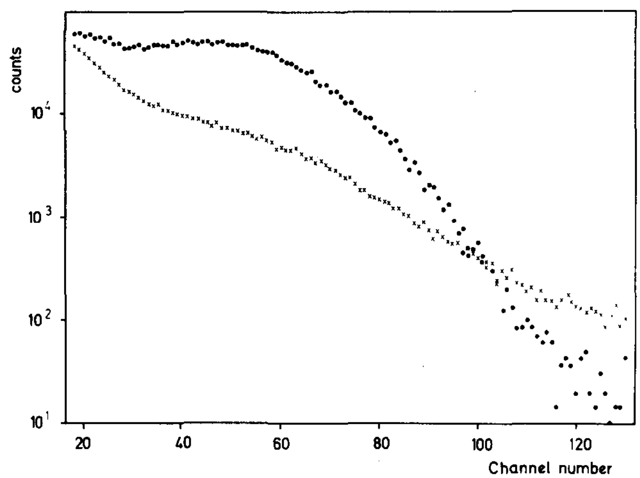

FIG. 1. Dark noise pulse height distribution n_{0i} and sensitivity pulse height distribution ϵ_i of an EMI 6256 A photomultiplier at about 240 K. ×—n_{0i}; ●—ϵ_i.

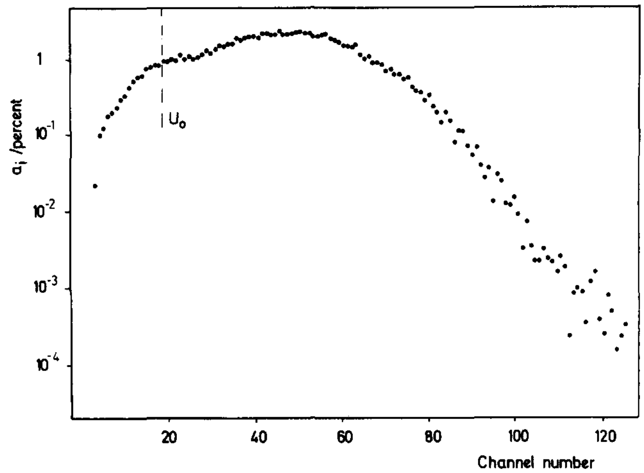

FIG. 2. The optimized weighting factors a_i obtained from the data given in Fig. 1. U_0 indicates the optimized triggering level of the conventional counting technique.

where z is the channel number corresponding to the triggering level U_0. It may be shown easily that the relative error of this second method is always larger, even if we confine ourselves to the region above the integral triggering level. Only in a special case, namely, if ϵ_i/n_{0i} is independent of U_i, i.e., if the pulse height spectrum of the dark pulses is proportional to that of the photopulses, the same error is obtained in both cases. In this case, no optimum triggering level U_0 exists. The gain of information which can be obtained by our method depends on the pulse height distributions of ϵ_i and n_{0i}. In the following, this gain will be demonstrated for a special example.

Our method of optimization was tested with a photomultiplier (type EMI 6256 A) which was cooled down to about 240 K. The luminescence of a very weakly excited diode was registered in a long term measurement.

Figure 1 shows the dark noise pulse height distribution n_{0i} for the tube and the sensitivity pulse height distribution ϵ_i. With these data the a_i were computed as well as the integral triggering level U_0 used in the conventional counting technique. The position of U_0 compared with the a_i curve is shown in Fig. 2 and demonstrates that some information is wasted normally. By means of the Eqs. (10) and (11) the ratio of the S/N ratio $M/\Delta M$ (optimized method) to the S/N ratio $M'/\Delta M'$ (normal method) was determined to be 1.20. This ratio was now verified experimentally. Forty equivalent measurements were made with the same intensity I. From each measurement there was computed a value M using our optimized method as well as a value M' using the conventional method. According to the formula

$$(\Delta M)^2=\frac{1}{39}\cdot\sum_{i=1}^{40}(M_i-\bar{M})^2, \tag{12}$$

the mean error was determined in both cases. With these data the ratio of the two S/N ratios was found to be

$$(M/\Delta M)/(M'/\Delta M')=1.34. \tag{13}$$

The experimental value shows a still better ratio than the theoretical one. The difference may depend on the fact that the conventional method reacts very sensitively to fluctuations of the triggering level, the amplification, and the multiplier supply voltage, respectively. We cannot exclude such fluctuations, because the total duration of the measurement was more than 10 days. It is a further advantage of our measuring method that it is less sensitive to such fluctuations. If a certain limit of error has to be reached, the proposed method shortens the necessary measuring time by a factor of nearly 2 in the given example.

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[1] See, for example, G. A. Morton, Appl. Opt. 7, 1 (1968); R. G. Tull, ibid. 7, 2023 (1968) and extensive bibliography associated with each.

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A Sensitive Calorimetric Method for Scanning Measurements of the Optical Absorption of Metals and Alloys*

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We have developed a sensitive calorimetric method for measuring the absorptivity of metals in the visible and infrared. The short thermal time constant of the system, typically 25 msec, has allowed us to use phase sensitive detection and continuous wavelength scanning. The absolute accuracy of this method was five times better than the best accuracy obtained in reflectivity measurements. The system has also been used to measure the difference between the absorptivities of pure and dilute alloy samples.

INTRODUCTION

THE optical properties of metals can be obtained by measuring the reflectivity R. This works well as long as the reflectivity differs substantially from unity. However, for metals in the infrared the quantity of physical interest is the absorptivity $A=1-R$. To obtain A accurately from a reflectivity measurement requires a measurement of R of extremely high precision, and systematic errors make this difficult. Bennett *et al.*[1] have developed a reasonably precise reflectometer for point-to-point measurements in the visible and infrared. A continuously operating instrument has been subsequently developed by Beaglehole.[2] These instruments measure the reflected light, and systematic errors restrict the absolute accuracy to about one part in 10^3. In many cases this is insufficient accuracy in the high reflectivity regions.

We have built a simple calorimetric system which measured the energy absorbed in the sample by determining the sample's rise in temperature. Calorimetric measurements are not new. They have been used at He temperatures by Biondi,[3] Biondi and Rayne,[4] and by Bos and Lynch,[5] and at room temperatures (with less sensitivity) by Shröder and Önengüt.[6] These methods were slow, having a thermal time constant of 10 sec or more, and therefore subject to thermal drift, and high intensities of incident light were needed to obtain measurable temperature rises with a corresponding loss in wavelength resolution.

Our rapid calorimetric technique was based upon the extremely sensitive Ge bolometer as the temperature sensor, and samples of low thermal mass. We used thin films evaporated onto thin mica substrates as samples. This reduced the thermal mass of the composite sample–bolometer system to a value that allowed the thermal time constant to be low enough to use periodic illumination of the sample and phase sensitive detection, and to eliminate thermal drift.

We have used the system to study the absorption of gold and gold–iron alloys (0.5–5 at.% iron impurities)[7] in the range 0.3–0.85 μ.

Let us briefly consider the thermal properties of our sample and bolometer. An extensive review of the detection properties of bolometers has been given by Smith *et al.*[8] Figure 1 shows (a) the thermal circuit of a body of thermal mass C_T connected to the surroundings by a thermal resistance R_T and (b) its electrical analogy. Applying an

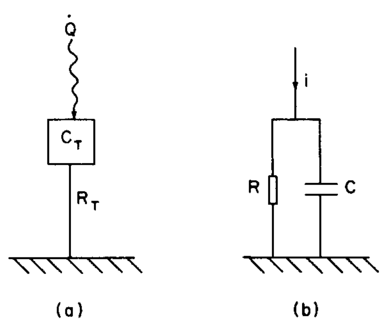

FIG. 1. (a) Thermal circuit of the bolometer and sample, and (b) its electrical analogy.